A Unified Framework for Computer-Aided Noise Analysis of Linear and Nonlinear Microwave Circuits

S. Heinen, J. Kunisch, and I. Wolff, FELLOW, IEEE

Department of Electrical Engineering and Sonderforschungsbereich 254
Duisburg University, Bismarckstr. 69, D-4100 Duisburg, FRG

Abstract

This paper introduces a new unified theoretical concept for noise analysis in analog and microwave circuits. Based on the adjoint system approach an analysis technique for general purpose CAD-applications is presented. The algorithm is easy to be implemented into existing CAD-tools. Moreover, it exploits the advantages of sparse matrix techniques.

Introduction

Noise analysis of analog circuits has been a major research topic for the last fifty years. The theories developed up to now mainly deal with the analysis of twoport circuits based on correlation matrices to achieve a complete characterization of the noise behavior of the circuit. Basically, the analysis is carried out by rebuilding the circuit by twoport interconnections, where the signal transfer and correlation matrices are calculated step by step [1]. This technique tends to get very complicated if the circuit is large. It seems to be nearly impossible to handle e.g. a complete operational amplifier in this way. In the early seventies a new technique for large circuits with uncorrelated noise sources was introduced in [3], which is restricted to the computation of noise voltages. However, considering e.g. a microwave amplifier, this algorithm cannot give a complete description of the noise behavior in terms of the four twoport noise parameters. In [2] the theory of correlation matrices for twoport networks is expanded to n-port networks. This generalization leads to a nodal based noise analysis, which can handle arbitrary linear circuits. However, this method is inefficient, because it requires the inverse of the complete nodal admittance matrix of the circuit, even if in many conventional microwave applications this causes no real problem, due to the relatively small circuit sizes.

In the area of nonlinear circuits, noise analysis has been restricted to a few specialized circuits up to now. Only in [11] an algorithm based on a piecewise harmonic balance technique and the noise description by n-port correlation matrices is discussed. Due to the concept of circuit partitioning used in the piecewise harmonic balance technique this algorithm is quite complicated and inefficient, too.

The work described here combines the advantages of the techniques used in [1, 2, 3, 11] to a unified theoretical framework. This yields an algorithm, which can handle circuits of arbitrary topologies. Moreover, it is easy to be implemented into existing simulators, because only a modified forward and backward substitution procedure is required plus a description of the noise current sources of the devices. As an example, in a linear SPICE-analysis of an amplifier only an additional forward and backward substitution for each port, two 2x2 matrix multiplications for each transistor, and the outer product of two 2-dimensional vectors for a resistor are needed.

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The Algorithm

For the sake of simplicity, we will first describe the algorithm for the case of linear circuits which are subject to a nodal analysis. This is no restriction because the application to the modified nodal analysis and the nonlinear case is straight forward. The modifications needed for the application to the nonlinear case will be discussed later on.

The aim of noise analysis is to replace the circuit's internal noise sources, which may or may not be correlated, by a set of equivalent correlated noise sources at the ports of the circuit. The presented algorithm is based on the adjoint system approach as described in [3, 4], which is used to calculate the transfer functions from the internal noise sources to the port noise sources. Therefore the sensitivities of the port voltages with respect to the internal current sources have to be calculated.

For a linear circuit, the nodal equations are given by

$$\mathbf{Y}\mathbf{U} = \mathbf{I}_q. \tag{1}$$

Assuming port i to consist of the nodes i and i', the voltage at this port can be expressed as

$$U_i = (\mathbf{e}_i - \mathbf{e}_{i'})\mathbf{U} = \mathbf{d}_i^T\mathbf{U}. \tag{2}$$

The resulting adjoint system

$$\mathbf{Y}^T\hat{\mathbf{U}}_i = \mathbf{d}_i I_N, \tag{3}$$

can be solved for the adjoint vector $\hat{\mathbf{U}}_i$, where I_n denotes a current of 1 A for unit consistence. The contribution of an internal source I_j to the voltage at port i is then given by

$$U_{ij} = Z_{ij}I_j, \tag{4}$$

where

$$Z_{ij} = \hat{\mathbf{U}}_i^T\frac{(\mathbf{e}_j - \mathbf{e}_{j'})}{I_N} \tag{5}$$

is the transfer function from the internal source to the output. Considering a network with n ports and a noisy device located anywhere in the network, where the noise properties of the device are described by a correlation matrix $\mathbf{C}^Y$ of l noise current sources, the relationship between the vector of equivalent noise voltage sources at the ports and the vector of internal noise current sources

$$\mathbf{U}_{port} = (U_1, U_2, U_3, \cdots, U_n)^T, \tag{6}$$

$$\mathbf{I}_{device} = (I_1, I_2, I_3, \cdots, I_l)^T, \tag{7}$$

is given by

$$\mathbf{U}_{port} = \mathbf{Z}\mathbf{I}_{device}. \tag{8}$$

The elements of the matrix of transfer impedances

$$\mathbf{Z} = [Z_{ij}] \tag{9}$$

are calculated from eq. (5). Using the above definitions, the impedance representation of the n-port correlation matrix is given by

$$\mathbf{C}^Z_{port} = \left\langle \mathbf{U}_{port}\mathbf{U}^{*T}_{port} \right\rangle = \mathbf{Z}\mathbf{C}^Y_{device}\mathbf{Z}^{*T}. \tag{10}$$

In eq. (10) no assumptions concerning the nature of the device's noise sources have been made. Therefore the device may contain any number of correlated or uncorrelated noise current sources. If there is more than one noisy device in the network, the port correlation matrix is a superposition of the individual contributions calculated from eq. (10). Moreover, the required figures of merit may be calculated starting from this correlation matrix.

Application to Modelling

For verification and comparison purposes, the described algorithm and the nodal based technique given in [2] were implemented, using standard algorithms for matrix inversion and factorization. In Fig. 1 the computation times for test circuits, with two ports and n nodes are given. The circuits are MESFET equivalent circuits starting from a very simple one with 2 nodes and finishing with a complete one with 10 nodes for microwave applications. The comparison of the computation times depends strongly on the implementation and the used computer. However, our algorithm exhibits the better performance and it's gain in computation time grows with the number of nodes. As a result of the MMIC-technology, the complexity of microwave circuits grows and new CAD-techniques are needed, which can handle large circuits. The presented algorithm combined with an appropriate sparse matrix method may be able to handle circuits with even hundreds of nodes. An other potential application of the presented technique is the extraction of the internal noise parameters R, P and C of the intrinsic MESFET.

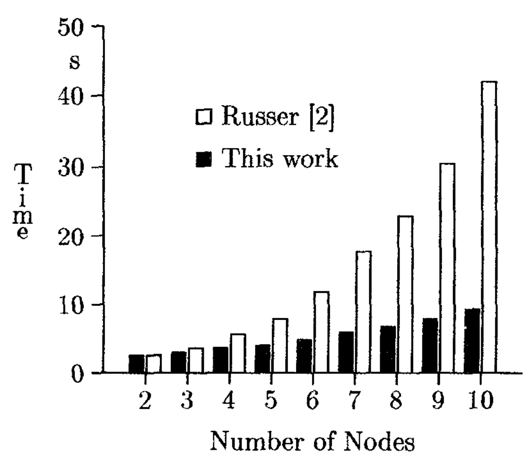

Figure 1: Comparison of computation times.

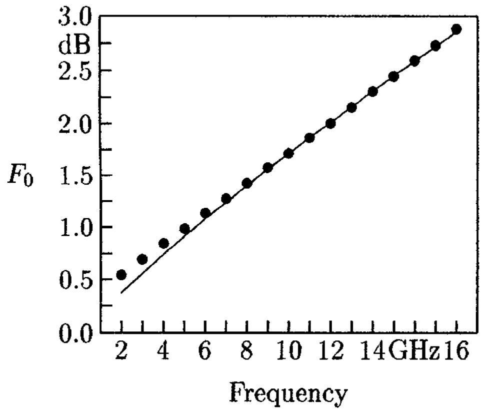

Figure 2: Minimum noise figure of an inhouse MESFET, ··· measured, — computed from extracted noise parameters

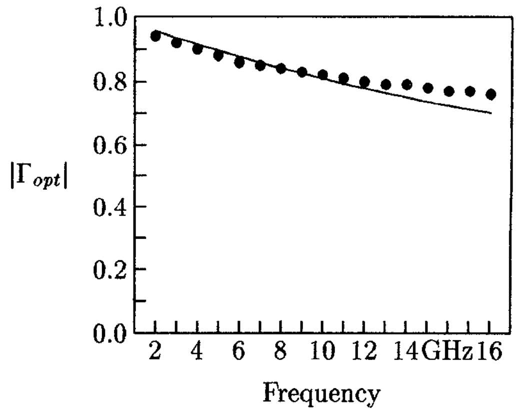

Figure 3: Magnitude of Γ_{opt} of an inhouse MESFET, ··· measured, — computed from extracted noise parameters.

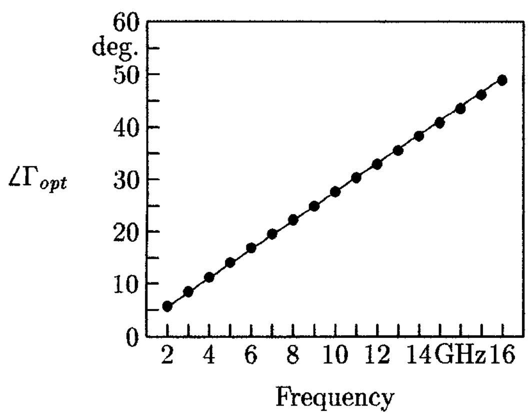

Figure 4: Angle of Γ_{opt} of an inhouse MESFET, ··· measured, — computed from extracted noise parameters.

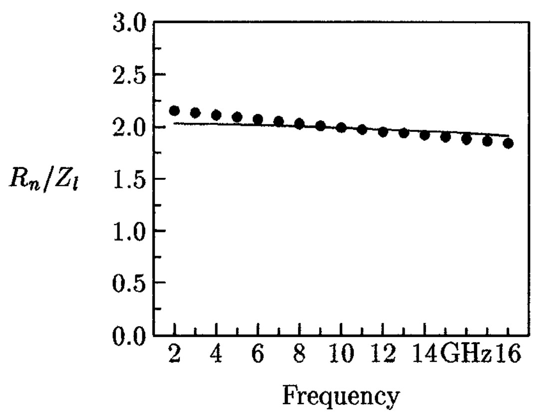

Figure 5: Equivalent noise resistance R_n of an inhouse MESFET, ··· measured, — computed from extracted noise parameters.

The results shown in Figs. 2-5 are computed using the extracted internal noise parameters. The extraction was done by reversing the described algorithm and averaging over the frequency range. It is worth noting that the deviation of the computed results is within the measurement accuracy. Therefore it seems to be resonable for CAD-application to characterize the noise behavior of a MESFET only by the three quantities R, P and C.

Application to Nonlinear Circuits

As shown in [5] the presented method is applicable for nonlinear circuits in combination with the harmonic balance method, as mentioned before. Due to it's nature noise is a small signal, which does not disturb the large signal steady state of the circuit. The Jacobian, used in the Newton-Raphson method [7], describes the transfer properties of the noise in the circuit under large signal excitation. Therefore it is directly related to the nodal admittance matrix and the presented technique can be used. It should be noted that in the nonlinear case noise sources of solid state devices are modulated by the large signal steady state and therefore some additional steps are needed to handle these modulated noise sources. Moreover, the dimension of the Jacobian is determined by the product of the number of considered harmonics and the number of nodes, resulting in a high dimension even for small circuit sizes. Especially in this case the advantages of the described technique may be utilized. It should be noted that a Jacobian calculated using the almost-periodic Fourier-transform (APFT) [8] method does not exhibit the required accuracy, due to the limited dynamic range of the APFT method [6, 9].

Conclusions

An algorithm for the nodal based noise analysis of linear and nonlinear analog circuits has been presented. The method eliminates the need of high dimensional matrix inversions, whichs occurs in conventional noise analysis algorithms in the case of large scale circuits or nonlinear circuits. Moreover, there are a lot of potential applications for the adjoint nodal noise analysis in the area of noise parameter extraction for a noisy nonlinear FET model [11], where the noise parameters R,P and C of the intrinsic FET are needed. In this paper we can not touch all the properties of the presented noise analysis technique and the explanation has been restricted to special combinations of noise sources (internal current and external voltage sources), but the method is not limited to this. In the authors' opinion the noise analysis technique based on the adjoint system concept is well suited for CAD-applications.

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